At the higher angle of attack, as shown in Fig. 2, the onset of intermittent separation seems to have occurred at about x/c of 0.65; the flow having reached full separation somewhere between x/c of 0.75 and x/c of 0.80, as can be inferred from velocity profiles as shown in Fig. 4a. This observation is further substantiated by the surface skin friction results as shown in Fig. 4b. Extrapolating skin friction data, one obtains the condition for full separation at x/c of 0.78.

Figure 5 depicts the comparison of the present results with the separation criteria as discussed by Sandborn and Kline.[9] The latter are based on the examination of several turbulent separation velocity profiles. For intermittent separation they have proposed the relation

$$H = 1 + (1 - \delta^*/\delta_{0.995})^{-1} \tag{1}$$

The present data are in good agreement with these criteria. At the lower angle of attack ($\alpha = 10.3$ deg), the velocity profiles indicate the presence of the intermittent separation at x/c of 0.91. The integral properties corresponding to this profile correlate well with the intermittent separation criterion. Although no velocity profile data were taken between x/c of 0.75 and 0.80, for $\alpha = 14.4$ deg, interpolation between these two points places x/c of 0.78 near the fully-developed separation line. Furthermore, the present data follow the results of Sandborn and Liu[10] more closely than the separation correlation of Perry and Schofield.[11] The data presented by Sandborn and Liu[10] and the present results are for the large change in curvature, unlike the Perry and Schofields' data.[11] This indicates that the surface curvature plays an important role in determining the path ($H\nu/s\delta^*/\delta_{0.995}$) by which a boundary layer separates.

Downstream of separation the outer region velocity profiles behave similar to a two-dimensional mixing layer. Figure 6a shows the typical fully separated flow profiles for $\alpha = 14.4$ and 18.4 deg. If the outer region mean velocity profile, between the points 1 and 2 as shown in Fig. 6b, is expressed in terms of the following parameters:

$$\eta = (y - y_2)/(y_c - y_2) \tag{2}$$

and

$$f(\eta) = (1 - u/U_e)/(1 - u_2/U_e) \tag{3}$$

where, y_c is the distance from the airfoil surface, such that

$$u_c = 0.50U_e[1 + (u_2/u_e)] \tag{4}$$

then the chordwise development of the mean velocity is found to be reducible to a universal curve.

Conclusions

The separated flow velocity profiles, skin friction distributions, and boundary-layer integral properties are presented for a 17% thick NASA GA(W)-1 airfoil. The integral properties corresponding to separation agree well with the separation criteria of Sandborn and Kline.[9] Downstream of the separation the outer region velocity profiles show some similarity and the chordwise development of the mean velocity is found to be reducible to a universal curve.

Acknowledgments

This work was supported under a NASA Contract NAS1-13985. The authors wish to thank Mr. Harry Morgan, the contract monitor, for his support and encouragement during the course of this investigation. The authors also would like to thank Dr. Jark C. Lau, of Kimberly Clark Corporation, for reviewing the manuscript and for his invaluable suggestions and encouragement. Thanks are also due to Drs. Zalman Lavan and W. M. Worek of IIT for their encouragement and support in preparation of this manuscript.

References

[1] Kline, S. J., "Some New Conceptions of the Mechanism of Stall in the Turbulent Boundary Layers," *Journal of the Aeronautical Sciences*, Vol. 24, Feb. 1957, pp. 470-471.

[2] Simpson, R. L., Strickland, J. H., and Barr, P. W., "Turbulent Boundary Layer Near Separation," *Journal of Fluid Mechanics*, Vol. 79, March 1977, pp. 553-594.

[3] Chu, J. and Young, A. D., "Characteristics of a Separated Incompressible Turbulent Boundary Layer," AGARD CP-168, Paper 13, 1976.

[4] Samuel, A. E. and Joubert, P. N., "A Boundary Layer Developing in an Increasingly Adverse Pressure Gradient," *Journal of Fluid Mechanics*, Vol. 66, Nov. 1974, pp. 481-505.

[5] Coles, D. and Hirst, E., *Compilation of Turbulent Boundary Layers*, Data Compilation, AFOSR-IFP, Stanford, Vol. 2, 1968.

[6] Sovran, G., *Compilation of Turbulent Boundary Layers*, AFOSR-IFP, Stanford, 1968, pp. 447-455.

[7] Goradia, S. H., Mehta, J. M., and Shrewsburry, G. S., "Analysis of the Separated Boundary Layer Flow on the Surface and in the Wake of Blunt Trailing Edge Airfoils," NASA-CR 145202, April 1977.

[8] Stevens, W. A., Goradia, S. H., and Braden, J. A., "Mathematical Model for Two Dimensional Multi-component Airfoils in Viscous Flow," NASA CR-1845, Feb. 1971.

[9] Sandborn, V. A. and Kline, S. J., "Flow Models in Boundary Layer Stall Inception," *Journal of Basic Engineering*, Transactions of the ASME, Vol. 83, May 1961, pp. 317-327.

[10] Sandborn, V. A. and Liu, C. Y., "On Turbulent Boundary Layer Separation," *Journal of Fluid Mechanics*, Vol. 32, May 1968, pp. 293-304.

[11] Perry, A. E. and Schofield, W. H., "Mean Velocity and Shear Stress Distributions in Turbulent Boundary Layers," *Physics of Fluids*, Vol. 16, Dec. 1973, pp. 2068-2074.

Separation Criteria for Three-Dimensional Boundary-Layer Calculations

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Introduction

THE thin boundary-layer equations solved with a prescribed pressure distribution are often used to obtain a prediction of the three-dimensional viscous flow about streamlined bodies such as ship hulls or aircraft components. In many cases, the location and nature of the separation are also deduced from the solution. The question arises whether this is a correct deduction. It is possible that the calculated results are misleading in this respect, since in at least some situations the boundary-layer equations are singular at the separation line. This means that no valid solution can be obtained in the whole region of influence of this singularity. In the following, this is designated as the "forbidden region."

Related to this is the concept of inaccessibility. According to Cebeci et al.,[1] a point in the boundary layer is inaccessible from the forward stagnation point if the velocity field there cannot be computed in terms of the initial conditions at the

Received March 18, 1983; revision received June 7, 1983.

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forward stagnation point and the boundary conditions on the body and in the external flow. This is true not only for all points in a closed separation region, but also for the points in their region of influence. In this case the forbidden region coincides with the inaccessible region, but in other cases it may be larger: with certain formulations a discontinuous weak solution of the differential equations might be found; however, such a solution would be affected by the discontinuity and therefore be invalid in its region of influence. Thus, this region would strictly be accessible, but forbidden.

From a practical point of view it is, therefore, quite important to detect in a numerical solution the occurrence of a separation singularity and the associated forbidden region, such that this part of the calculated results can be discarded. To this end separation criteria based on both the analytical behavior of the boundary-layer equations and the properties of the numerical method are needed.

In two-dimensional steady flows the prescription of separation criteria is of little concern. The separation point is clearly defined by the reversal of the streamwise velocity component close to the wall and the vanishing of the wall shear stress. The solution of the boundary-layer equations with a prescribed pressure gradient contains, in laminar flows, a Goldstein singularity that leads to an infinite normal velocity and displacement thickness at the separation point. In a numerical calculation, the approach of this singularity is apparent by the deteriorating convergence of the iterative procedure used for solving the nonlinear terms in the momentum equations. Thus, there is generally no risk of missing a two-dimensional steady separation in a calculation.

For three-dimensional flows, neither the flow reversal nor the vanishing of the wall shear stress can be used as general separation criteria, particularly for open separation. Wang[2,3] and Tobak and Peake[4] argue that an open separation line does not start in a singular point (defined as a point of zero skin friction). Other types of singular behavior are not considered by these authors, and are even excluded by the basic assumption in Ref. 4 that the wall shear stress is a continuous vector field. However, according to Cousteix and Houdeville,[5] discontinuities may occur in the solution of the boundary-layer equations with a prescribed pressure gradient, corresponding with a focusing of limiting streamlines.

The same case investigated by Wang,[2] the flow past a prolate spheroid at incidence, has been calculated by Cebeci et al.[1] At the windward boundary of the wedge-shaped region of separated flow, they found strong evidence that the wall shear stress was singular, although nonzero.

These investigations of the analytical aspect of three-dimensional separation prediction indicate that singularities can occur. However, the second aspect on which separation criteria should be based, the behavior of the numerical solution in the vicinity of such singularities, has been given little attention. Its importance is suggested by the considerably different separation positions obtained by Wang[2] and Cebeci et al.[1] with different numerical methods. However, in this case the numerical effects are obscured by the uncertainty about the analytical flow structure. Therefore, in the present study calculations have been made for a case in which a singularity is known to occur.

Numerical Study

The method used was the boundary-layer program developed at the Netherlands Ship Model Basin, details of which are given in Ref. 6. It is a finite difference method solving the continuity equation and the two momentum equations in directions parallel to the wall in an orthogonal or nonorthogonal coordinate system. For turbulent flows the Cebeci and Smith turbulence model is applied. Dependent on the sign of the crosswise velocity component, the Crank-Nicolson or the Krause zig-zag finite difference scheme is used, such that the zones of influence are properly taken into account.

This method was applied to the turbulent boundary layer on a 35 deg swept wing of infinite span, corresponding with an experiment by the NLR.[7] In this case an orthogonal grid was used with the x direction chordwise and the z axis along the leading edge. The adverse pressure gradient induces separation at a distance of 1.07 m from the leading edge in the experiment. Here, the chordwise velocity and shear stress components vanish at the wall ($u = \tau_x = 0$) and the limiting streamlines are parallel to the trailing edge.

The usual method of calculating this case exploits its quasi-two-dimensional character: all spanwise derivatives are zero, and the solution is found by marching along a single line in the chordwise direction. The spanwise flow plays no role in the solution for the chordwise shear stress τ_x, except by a weak coupling through the turbulence model. τ_x therefore contains the same singularity at separation as in a two-dimensional flow. Indeed, the separation is easily detected by the nearly simultaneous occurrence of reversal of the chordwise velocity u and the nonconvergence of the iterative solution. This happens at 1.207 m from the leading edge, further aft than the experimental separation; similar deficiencies have been found with other calculation methods. The region beyond the calculated separation line remains inaccessible.

Without dropping the spanwise derivative terms at the outset, the calculation becomes effectively three-dimensional. It thus seems possible to find a solution in the separated region by marching spanwise. To this end, initial data along a chord $z = 0$ and boundary conditions near the leading and trailing edges and the boundary-layer edge were derived from the experimental data. So at $z = 0$, there is no singularity and the calculation easily proceeds on both sides of the line $\tau_x = 0$. This is what one expects when calculating a flow with an open separation line that is roughly aligned with the marching direction, a case frequently encountered in practice. How will the emergence of a singularity be detected in such cases?

One would suppose the results to settle to a constant infinite swept-wing solution for large z. But, as illustrated by Fig. 1, a spanwise invariant solution is found only in the attached flow region; beyond the line $\tau_x = 0$ the boundary layer grows rapidly. The completely different development on both sides of the separation line results in excessive values of the crosswise derivatives here, and eventually the iterative solution process fails to converge. Meaningless results are obtained and within a few steps the calculation breaks down by the occurrence of arithmetic errors.

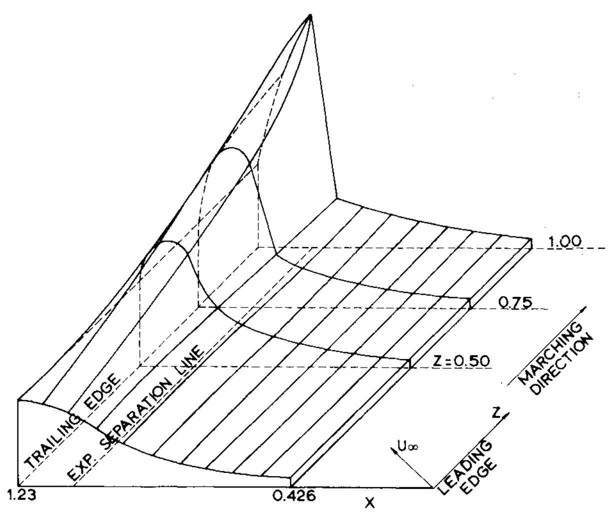

Fig. 1 Calculated momentum thickness of infinite swept-wing boundary layer showing the emergence of a discontinuity in spanwise marching calculation.

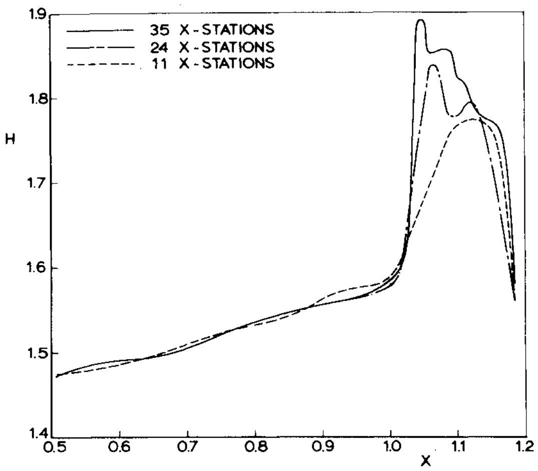

Fig. 2 Grid dependence of shape factor H at $z = 0.75$ m.

Table 1 Calculation parameters and results (chord length 1.23 m, unit Reynolds number 2.42×10^6)

No. of chordwise grid points	11	24	35
Δx near separation line, m	0.08	0.02	0.01
Breakdown at z, m	1.300	0.984	0.855
at x, m	1.081	1.040	1.040

This breakdown is often used as an indication of separation. However, the forbidden region may not be identified simply with the zone of influence of the point where breakdown occurs. The continuous growth of the displacement thickness beyond the separation line at smaller z values is at variance with the infinite swept-wing behavior and already indicates the effect of a singularity. Therefore, this part of the results should also be discarded. But, in general three-dimensional cases one does not always have the prior knowledge required for judging the physical meaning of the solution; thus, other criteria must be applied.

The same calculation has been repeated with various chordwise grid spacings Δx, ranging from 0.08 to 0.01 m ($\simeq 0.16\delta$) near the separation line. The resulting breakdown positions are listed in Table 1. The smaller Δx is, the more clearly a discontinuity at the line $\tau_x = 0$ is formed, while the breakdown is shifted to smaller z values. The discontinuity is located remarkably close to the experimental separation line at $x = 1.07$ m. This suggests that the warning in Ref. 5 not to associate discontinuities with separation lines is perhaps too pessimistic.

With a fine grid, the breakdown occurs at a z position where a coarse-grid calculation does not give a clear indication of a singularity. In the case considered here, breakdown will eventually occur even with the coarsest mesh due to the persistence of the adverse pressure gradient. But, in general three-dimensional flows it may happen that the singularity is passed without being noticed, as appears from results shown in Ref. 8. Thus, unconsciously one obtains erroneous results in the forbidden region. This is illustrated in the present case by the strong grid dependence of the results beyond the line $\tau_x = 0$, particularly of the shape factor (Fig. 2).

The "surface transpiration velocity" $v_W = v(\delta) - \delta(\partial v / \partial y)_{\text{inviscid}}$, a measure of the strength of the viscous-inviscid interaction, shows a sharp, rapidly growing peak with the finest grids, which is much less pronounced with a coarse grid. It is unlikely that a viscous-inviscid interaction procedure can be made to converge in this way.

Conclusions

These results indicate that in the solution for the three-dimensional case investigated, which resembles an open separation, a discontinuity of the displacement thickness and other boundary-layer characteristics exists at the separation line. This discontinuity gives rise to a forbidden region in the sense that the solution of the boundary-layer equations in its region of influence can be physically meaningless. In a numerical calculation, results can sometimes be found in at least a part of this region, but these are severely grid dependent and can never be the basis of a successful iterative viscous-inviscid interaction calculation.

To recognize the forbidden region one needs a criterion that permits the detection of separation and associated singularities. In three-dimensional cases the breakdown of the calculation is an unreliable and insufficient criterion, as demonstrated above.

Based on the experiences gained we suggest that the grid dependence of the calculated solution be used as the main indicator of a singularity. When the occurrence of separation is suspected, even when no breakdown has taken place, a local grid refinement will show whether or not the results obtained represent a valid and regular solution. The extra effort is in most applications widely compensated for by the increased confidence in the solution and the information on the extent of the region of separated flow.

Acknowledgment

This work has been supported in part by the National Foundation for the Coordination of Maritime Research in the Netherlands (CMO) under Project CTEC 101-81.

References

[1] Cebeci, T., Khattab, A. K., and Stewartson, K., "Three-Dimensional Laminar Boundary Layers and the Ok of Accessibility," *Journal of Fluid Mechanics*, Vol. 107, 1981, pp. 57-87.

[2] Wang, K. C., "Boundary Layer Over a Blunt Body at Low Incidence with Circumferential Reversed Flow," *Journal of Fluid Mechanics*, Vol. 72, 1975, pp. 49-65.

[3] Wang, K. C., "New Developments in Open Separation," *Proceedings of the IUTAM Symposium on Three-Dimensional Turbulent Boundary Layers*, Springer-Verlag, Berlin, 1982, pp. 94-105.

[4] Tobak, M. and Peake, D. J., "Topology of Three-Dimensional Separated Flows," *Annual Review of Fluid Mechanics*, Vol. 14, Annual Reviews Inc., Palo Alto, Calif., 1982.

[5] Cousteix, J. and Houdeville, R., "Singularities in Three-Dimensional Boundary-Layer Calculations and Separation Phenomena," *AIAA Journal*, Vol. 19, Aug. 1981, pp. 976-985.

[6] Raven, H. C., "Calculation of the Boundary-Layer Flow around Three Ship Afterbodies," *International Shipbuilding Progress*, Vol. 27, Jan. 1980, pp. 10-29.

[7] Van den Berg, B., Elsenaar, A., Lindhout, J.P.F., and Wesseling, P., "Measurements in an Incompressible Three-Dimensional Turbulent Boundary Layer under Infinite Swept Wing Conditions, and a Comparison with Theory," *Journal of Fluid Mechanics*, Vol. 70, 1975, pp. 127-148.

[8] Larsson, L. (ed.), "Proceedings of the SSPA-ITTC Workshop on Ship Boundary Layers," Publications of the Swedish Maritime Research Center, Gothenburg, N. 90, 1980.